# Modelling in Curricula across the World

**Panel discussion**

**ICTMA-17, Nottingham**

**July 24, 2015**

ICTMA-17
NOTTINGHAM 2015

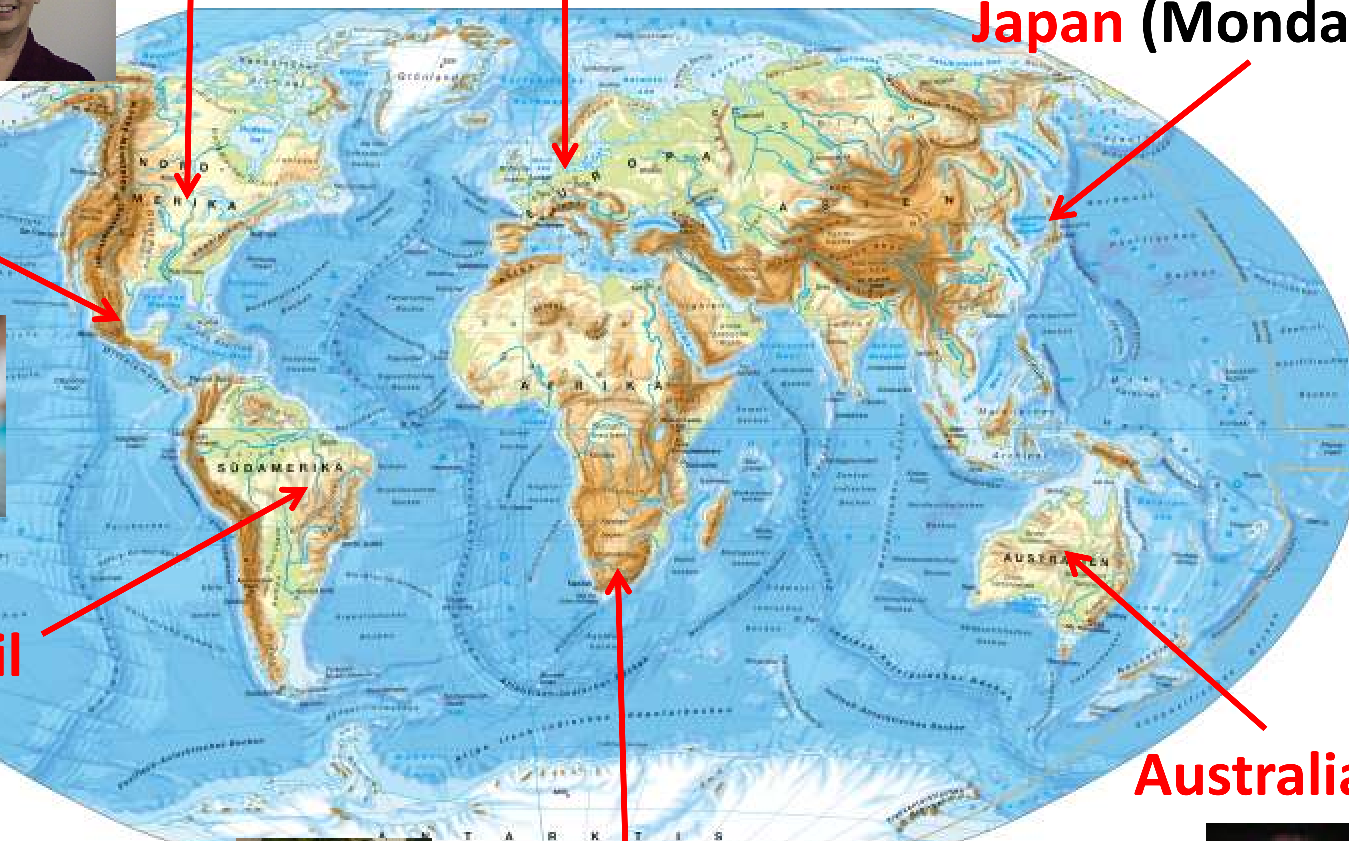
## Our journey:

USA

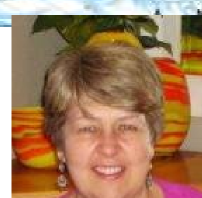
Netherlands & Norway

Japan (Monday)

Mexico




Brazil



South Africa

Australia

# Questions to the panelists:

- What is the role of modelling in the curriculum of your country (perhaps including other, e.g. neighbouring, countries if appropriate), including assessment?
- Are there big discrepancies to actual classroom practice?
- What role - if any - has the international modelling community played in the developments in your country, and which role could it play?

**Structure of the panel discussion:**

- Introduction (3 min)
- Each panelist: 8 min statements plus 2 min questions from the audience (50 min)
- Plenary discussion with the audience (25 min)
- Each panelist: 2 min final statements (10 min)
- Closing remarks (2 min)

# The Case of USA

# The Role of Modeling in the United States Curriculum

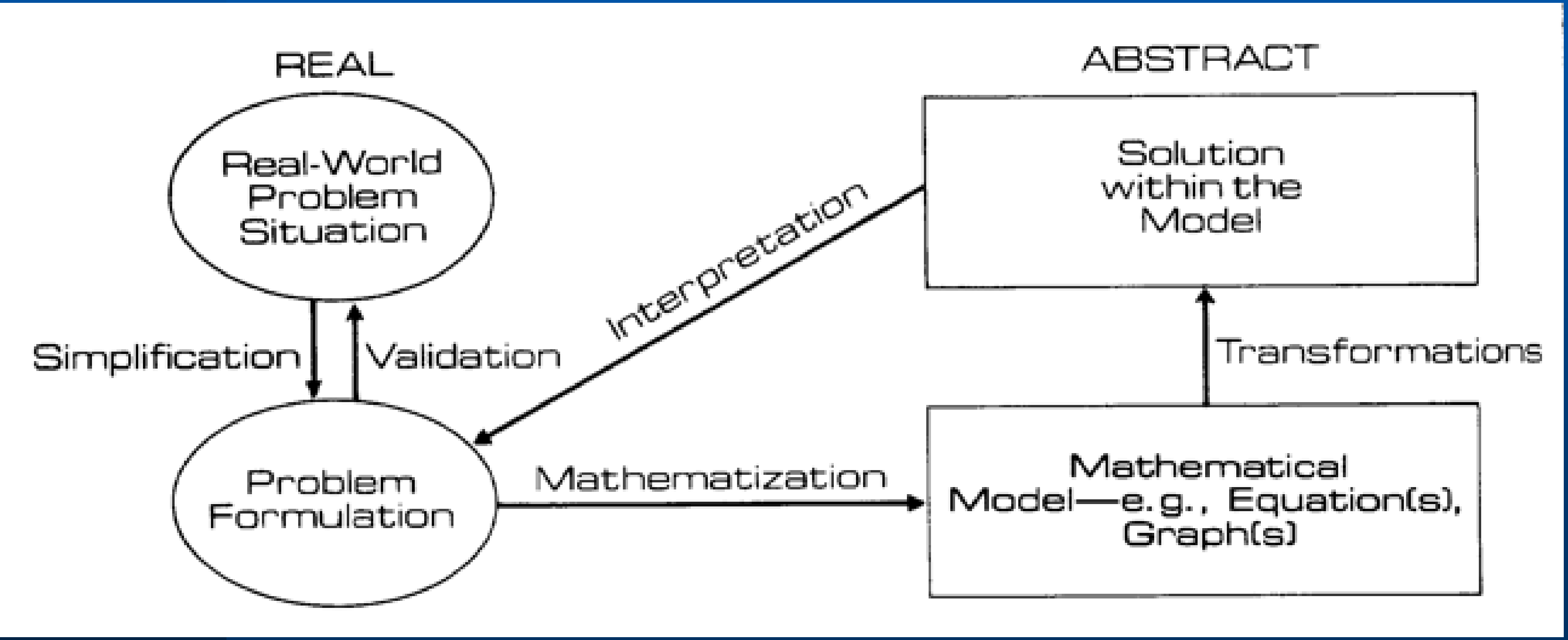
Helen M. Doerr
Syracuse University

# The Standards Documents

- NCTM 1989 Curriculum and Evaluations Standards for School Mathematics
- NCTM 2000 Principles and Standards for School Mathematics
- The Common Core State Standards for Mathematics 2010

# The 1989 standards

- Modeling was part of the "problem solving" standard



# In the 1989 standards

- One illustration of modeling was given in the document
- About the fairness of a two-player coin toss game
- Validation was suggested by playing the game or by computer simulation

# Following the 1989 Standards

- Substantial curriculum development effort funded by the National Science Foundation
- Resulted in curriculum materials to meet these new standards at all grade levels

# The 2000 Standards

- Modeling seems to disappear
- It is not mentioned in the "problem solving" standard
- BUT students are expected to "formulate, approach and solve problems"
- "problems that occur in real settings do not often arrive neatly packaged"

# The 2000 Standards

- The “representation” standard states that all students should be able to “use representations to model and interpret physical, social, and mathematical phenomena”
- “recognize common mathematical structure across contexts”
- “create and interpret models” from a range of contexts

# Curricular Materials

- Curricular materials continued to develop and became more widely adopted
- Two major projects:
  - CMP – Connected Mathematics Project for the middle grades
  - Core-Plus Mathematics Project for four years of high school

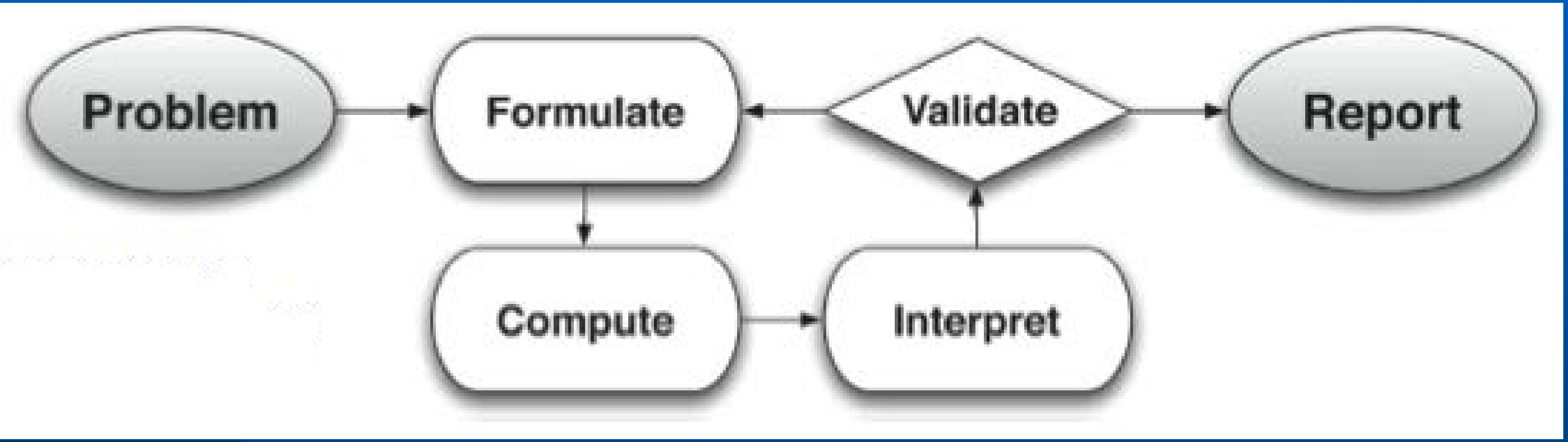
# Political Events

- The so-called "math wars"
- "No Child Left Behind" (2001) – assessment to improve national outcomes
- The "Race to the Top" (2009) for "rigorous and challenging standards and assessments"

# The Common Core State Standards in Mathematics

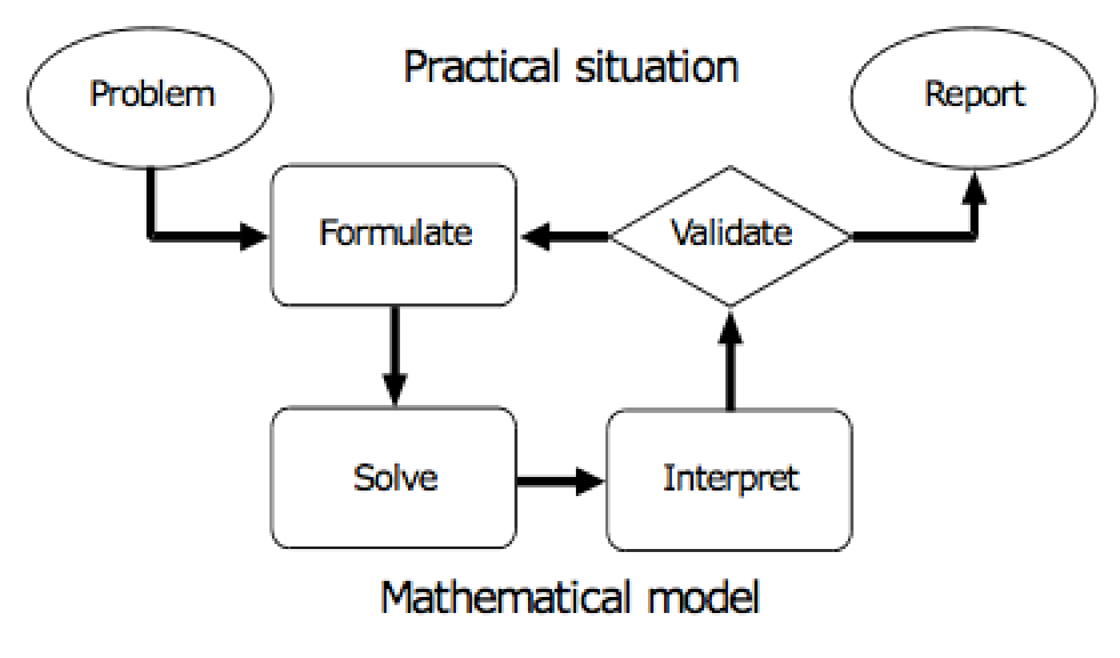
- The adoption of national standards by 45 states
- Modeling is a content standard in high school
- Modeling is a practice standard K-12

# Modeling in the Common Core State Standards




- Compute means to analyze, perform operations on relationships, and draw conclusions.

# Striking resemblance




Burkhardt (2007)

# Common Core State Standards

- Sparked new interest in modeling

As a practice:

- "can apply the mathematics they know to solve problems arising in everyday life, society and the workplace"
- make assumptions, identify quantities, draw conclusions, interpret results in context

# Common Core State Standards

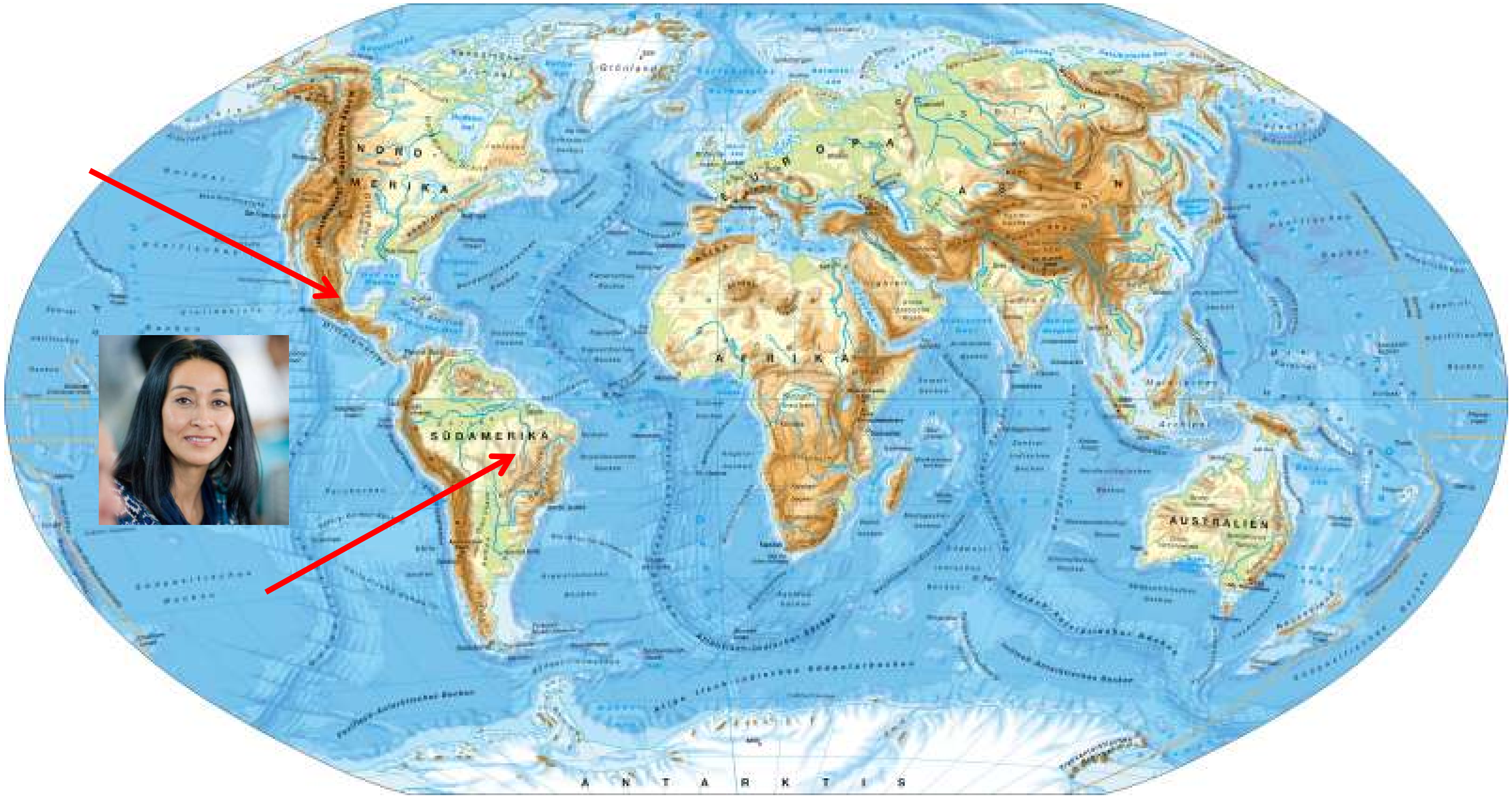
As content in high school:

- To analyze empirical situations, to understand them better, and to improve decisions
- Physical, economic public policy, social and everyday situations
- Technology for varying assumptions, exploring consequences, and comparing predictions with data

# Common Core State Standards

- Assessments are being put in place
- Very little curriculum available, much is being "modified"
- Common Core Standards are becoming somewhat more controversial

# The Cases of Mexico and Brazil



# Modelling in the curriculum in Latinoamerica

## Two cases: Mexico and Brazil

Ángeles Domíngues, Monterrey

# Mathematics Standards (1-6 grade)

They consist of:

1. Number Sense and Algebraic Thinking
2. Form, space and measurement
3. Information management
4. Attitude towards the study of mathematics

# Number Sense and Algebraic Thinking

Number sense and algebraic thinking refers to the most relevant late study of arithmetic and algebra:

- The **modeling situations** using the arithmetic language.
- The exploration of **arithmetic properties that will be generalized** with algebra in grades 7th to 9th and on.
- The **variety of ways to represent and perform** computations.

# Role of modelling in the curriculum

At the end of grade 9th, students are responsible for **construct new knowledge from their previous knowledge**, which implies:

- To **develop** and **validate** conjectures.
- To **ask** questions.
- To **communicate**, **analyze** and **interpret** solutions and procedures.
- To **argue** the validity of procedures and results.
- To find different ways to solve problems.
- To use different strategies and techniques efficiently.

# Teaching strategies

It is suggested for the study of mathematics, to use **sequences of problematic situations** to arouse the interest of students and invite them to **reflect**, to **find different ways to solve problems** and **make arguments that validate results**. At the same time, the situations should involve precisely the knowledge and skills to be develop.

# Big discrepancies to actual classroom practice

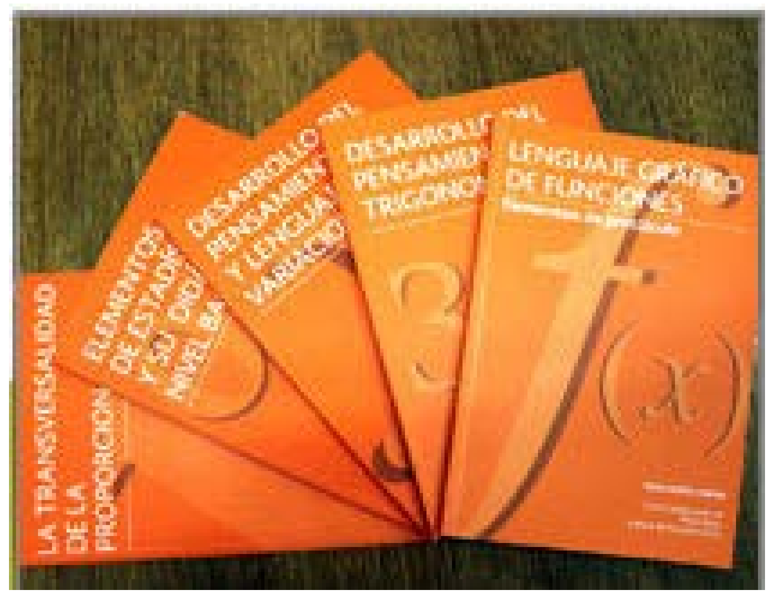
- Tremendous gap between what it is stated on paper (mathematics reform) and practice (in the classroom).

# Role of the modelling community

- Some efforts on bringing modelling into the classrooms

ECIT • EMAT Enseñanza de la Ciencia y las Matemáticas con Tecnología

- Few research group:
  - Mexico City
  - Monterrey
  - Quintana Roo



# History of modelling in Brazil

- **Aristides Barreto** proposes modelling for teaching (about 45 years ago).
- ICME-1 (1969) — Lyon (France)
- ICME-4 (1980) — Berkeley (USA)

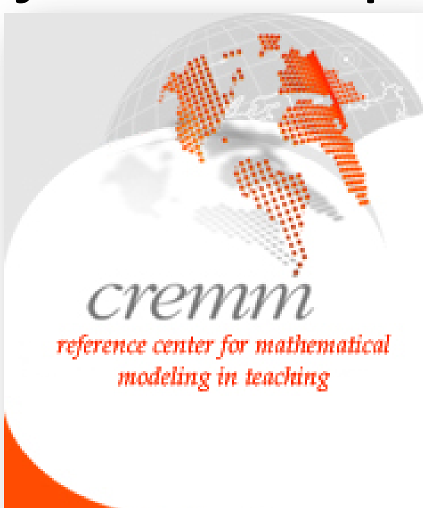
- **Ubiratan D'Ambrosio** – Modelling Modeling and Ethno-Mathematics (1975 - 1978)

http://www.furb.br/cremm/ingles/index.php

# History of modelling in Brazil

- 1980 **Rodney Bassanezi** follows his lead with great strength.

- 1986 **Maria Salett Biembengut**, joins the quest.




# Modelling in the curriculum

- Paraná State integrated modelling into the curriculum.
- Other states followed Paraná.
- Then, modelling gets integrated into the national curriculum for all grade levels.

- Moreover, the national high school exam (ENEM) integrates modelling into the assessment.

# PISA Results 2012

| | Mean score | Low achievers (Below Level 2) | Top performers (Level 5 or 6) |
|---|---|---|---|
| Chile | 423 | 51.5 | 1.6 |
| Mexico | 413 | 54.7 | 0.6 |
| Uruguay | 409 | 55.8 | 1.4 |
| Costa Rica | 407 | 59.9 | 0.6 |
| Brazil | 391 | 67.1 | 0.8 |
| Argentina | 338 | 66.5 | 0.3 |
| Colombia | 376 | 73.8 | 0.3 |
| Perú | 368 | 74.6 | 0.6 |

# The Case of South Africa


# The role of modelling in the South African curriculum

Helena Wessels, Stellenbosch University

# Intended school curriculum

- Primary school
  - Modelling not explicitly mentioned, problem solving a focal point
  - General and specific aims are
    - to identify and solve problems, making decisions using critical and creative thinking
    - to demonstrate understanding that the world is a set of related systems and that problem solving contexts do not exist in isolation
    - Cognitive development (problem solving, logical thought & reasoning)
  - Non-routine problems, higher-order understanding and processes, breaking up problems in constituent parts emphasized for upper primary school
  - Assessment of problem solving skills described in curriculum – example rubrics and scoring given

# Intended school curriculum

- High school
  - General and specific aims are
    - to identify and solve problems, making decisions using critical and creative thinking
    - to demonstrate understanding that the world is a set of related systems and that problem solving contexts do not exist in isolation
    - Cognitive development (problem solving, logical thought & reasoning)
  - Mathematical modeling as focal point: real-life problems should be included, realistic and not contrived. Contexts should include health, social, economic, cultural, scientific, political and environmental issues
  - Non-routine problems, higher order reasoning & processes emphasised
  - Assessment:
    - "Modelling as process should be included in all papers, thus contextual questions can be set on any topic"
    - Memo with model answers
    - Department of Basic Education discourages the use of rubrics

# Enacted school curriculum

- Pressure of Annual National Assessments – little time for real problem solving or modelling
- Teachers' understanding of problem solving –> textbook word problems
- Teachers' understanding of modelling and modelling problems rudimentary or non-existent
- Many high school teachers use textbook problems to comply

# Tertiary education

- Modelling courses or units in undergraduate and honours courses at a number of SA universities
- Assessment tasks; peer and lecturer assessment, group assessment
- Modelling topics in M and PhD studies at very few universities

# ICTMA role?

- Attending of ICTMA conferences, discussions with cognoscenti who also attended, joint projects with colleagues from other universities internationally, fostered the development and improvement of modelling at some of the universities and schools in SA
- Bank with modelling problems for all grades needed – ICTMA members contribute (coordinated by ICTMA special committee??)

# The Case of Australia


# Mathematical Modelling and Applications in Australia

Vince Geiger
Australian Catholic University

# Curriculum Landscape

# Curriculum Landscape

# Curriculum Landscape

# Curriculum Landscape

# Curriculum Landscape

# Australian Curriculum P-10

That all systems and schools recognise that, while mathematics can be taught in the context of mathematics lessons, the development of numeracy requires experience in the use of mathematics beyond the mathematics classroom, and hence requires an across the curriculum commitment.

(Council of Australian Governments, 2008, p. 7)

# Australian Curriculum P-10

**Numeracy as a General Capability**

Using mathematical skills across the curriculum both enriches the study of other learning areas and contributes to the development of a broader and deeper understanding of numeracy. Therefore, a commitment to numeracy development is an essential component of learning areas across the curriculum and a responsibility for all teachers. This requires that teachers:

- identify the specific numeracy demands of their learning area
- provide learning experiences and opportunities that support the application of students' general mathematical knowledge and skills
- use the language of numeracy in their teaching as appropriate.

# Australian Curriculum P-10

- Students become numerate as they develop the knowledge and skills to use mathematics confidently across all learning areas at school and in their lives more broadly. Numeracy involves students in recognising and understanding the role of mathematics in the world and having the dispositions and capacities to use mathematical knowledge and skills purposefully.

- Students develop numeracy capability as they learn to organise and interpret historical events and developments. Students learn to analyse numerical data to make meaning of the past, for example to understand cause and effect, and continuity and change. Students learn to use scaled timelines, including those involving negative and positive numbers, as well as calendars and dates to recall information on topics of historical significance and to illustrate the passing of time.

# Numeracy in the Australian Professional Standards for Teachers

- Standard 2.5 Literacy and numeracy strategies

Graduate career stage: Know and understand literacy and numeracy teaching strategies and their application in teaching areas.

Proficient career stage: Apply knowledge and understanding of effective teaching strategies to support students' literacy and numeracy achievement.

# Australian Curriculum Senior Secondary

- Very little mention of modelling (or numeracy) throughout the document but…
- Three out of four seniour mathematics subjects refer to modelling in the relevant assessment standards e.g.,

**Specialist mathematics**

*Concepts and Techniques*

develops, selects and applies mathematical models to routine and non-routine problems in a variety of contexts

*Reasoning and communication*

identifies and explains the validity and limitations of models used when developing solutions to routine and non-routine problems

# Modelling in Queensland

- Syllabus objective Mathematics B

1. Knowledge and procedures
2. Modelling and problem solving
3. Communication and justification

# Modelling in Queensland

***Modelling and problem solving***

The objectives of this category involve the uses of mathematics in which the students will model mathematical situations and constructs, solve problems and investigate situations mathematically across the range of subject matter in this syllabus. By the end of the course students should be able to:

- apply problem-solving strategies and procedures to identify problems to be solved, and interpret, clarify and analyse problems
- identify assumptions (and associated effects), parameters and/or variables during problem solving
- represent situations by using data to synthesise mathematical models and generate data from mathematical models
- analyse and interpret results in the context of problems to investigate the validity (including strengths and limitations) of mathematical arguments and models.

# Modelling in Queensland

**Principles of a balanced course**

- Application
- Technology
- Initiative
- Complexity

# Connect with Maths: Maths in Action

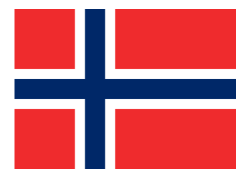

http://connectwith.mathsinaction.aamt.edu.au

# Maths in Action: Applications and Modelling Community


- Mathematica, computer-based maths and the new era of STEM
- Pulsars in the mathematics classroom
- What's happened to temperatures over the last century?
- Mathematica and the Barn Quilt Project
- Discovering Sustainability & Maths in a World Heritage Icon

# The Cases of Netherlands & Norway

# Modelling in the Dutch and Norwegian Curriculum



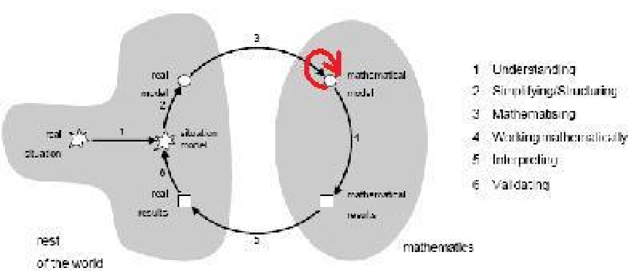
Pauline Vos, Agder University

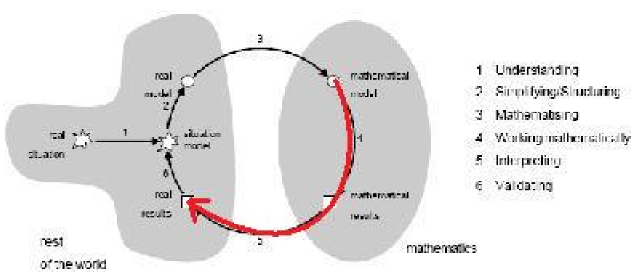
# What is the role of modelling in the curriculum of your country, including assessment?

- Realistic Mathematics Education was “born” in NL:
  - Realistic contexts to start from
    *“mathematics as to be useful”* (Hans Freudenthal)
    usefulness experienced during the learning process
  - Modelling as a vehicle for concept development
- Occasionally: modelling-for-modelling (day-long projects, A-lympiad)
- National exams: 100% of tasks is contextualised (exception: MathB 58-74%), 9-12 pages of text, “reproductive mathematizing”, ready-made models, never a real problem being solved

With what % does the value decrease each year?

Show that the value after 5 years is 40 euro.




## AANSCHAF NIEUWE FIETS

Celise gaat met de fiets naar haar werk. Ze heeft hiervoor een nieuwe fiets gekocht van € 530,-. Celise is van plan haar fiets na een aantal jaren in te ruilen. De fiets wordt elk jaar minder waard. Ze gebruikt de volgende formule als vuistregel voor het berekenen van de inruilwaarde:

$$w = 530 \times 0{,}6^t$$

Hierin is *w* de inruilwaarde van de fiets in euro en *t* het aantal jaren dat de fiets oud is.

Bij de vragen 17, 18 en 19 gaan we uit van bovenstaande formule.

**17** → Met hoeveel procent neemt de inruilwaarde van haar fiets elk jaar af?

**18** Celise wil weten wat de inruilwaarde van haar nieuwe fiets na 5 jaar zal zijn.
→ Laat zien dat de nieuwe fiets van Celise na 5 jaar ongeveer € 40,- waard is. Schrijf je berekening op.

[redacted] Windsnelheid en hoogte

(WisB1 and B12 havo2006)

Op een bepaalde dag is in Vlaardingen op verschillende hoogtes de windsnelheid gemeten. Uit de meetresultaten blijkt dat er bij benadering een lineair verband bestaat tussen de windsnelheid $W$ in m/s en de hoogte $h$ in meter voor hoogten tussen 10 en 80 meter (zie tabel 1). De formule $W = a \cdot h + b$ geeft dit lineaire verband.

tabel 1

| $h$ | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 |
|---|---|---|---|---|---|---|---|---|
| $W$ | 1,2 | 1,6 | 2,1 | 2,5 | 3,0 | 3,4 | 3,9 | 4,3 |

Calculate *a* and *b* for the formula.

4p **14** □ Bereken $a$ en $b$ met behulp van de gegevens in tabel 1.
Rond $a$ af op drie decimalen en $b$ op twee decimalen.

Onderzoek door weerkundigen naar windsnelheden op verschillende hoogtes en onder verschillende omstandigheden heeft opgeleverd dat het verband tussen windsnelheid en hoogte in het algemeen niet lineair is. Een betere formule is:

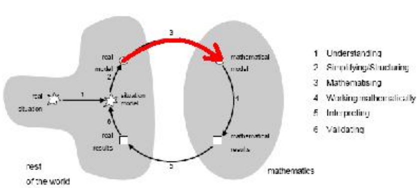
$$W = 5{,}76 \cdot m \cdot \log\left(\frac{h}{r}\right)$$

Hierin is:

- $W$ de windsnelheid (in m/s);
- $h$ de hoogte in meter waarop de windsnelheid wordt gemeten;
- $m$ een constante die afhangt van de wrijving tussen de luchtlagen;
- $r$ een constante die afhangt van de ruwheid van het terrein (hoge bomen beïnvloeden de windsnelheid anders dan grasland)

De formule is geldig tot hoogtes van ongeveer 100 meter.

In de praktijk wordt de windsnelheid op een hoogte van 10 meter gemeten. De waarde van $r$ op de meetplek is bekend zodat het getal $m$ met behulp van de formule berekend kan worden. Vervolgens kan met de gegeven formule de windsnelheid op andere hoogtes berekend worden.



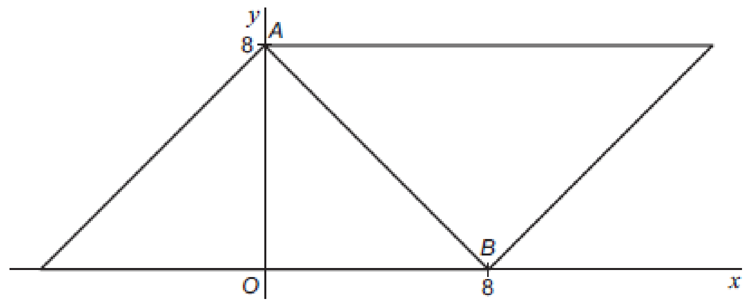

Boven open bouwland met $r = 0{,}12$ wordt de windsnelheid gemeten. Op 10 meter hoogte is deze windsnelheid 6,0 m/s.

5p **15** □ Bereken in deze situatie de windsnelheid op een hoogte van 60 meter.

## Bedekken

Een geodriehoek is een gelijkbenige rechthoekige driehoek. We plaatsen twee geodriehoeken met een lange zijde van 16 cm in een rechthoekig assenstelsel met eenheid 1 cm op de manier die in figuur 2 (verkleind) is getekend.
De top $A$ van de linker driehoek heeft de coördinaten (0, 8).
De top $B$ van de rechter driehoek heeft de coördinaten (8, 0).

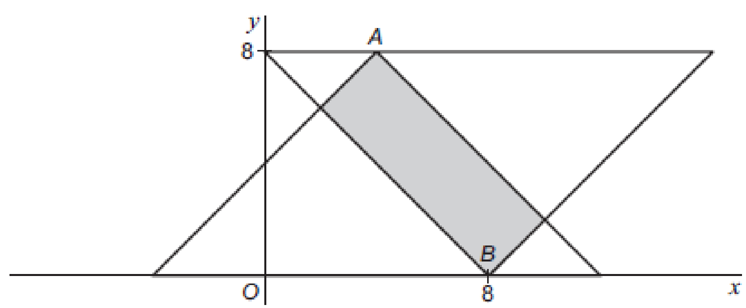
figuur 2




De linker driehoek begint op tijdstip $t = 0$ naar rechts te schuiven over de rechter driehoek met een snelheid van 1 cm/s. Daarbij wordt een gedeelte van de rechter driehoek door de linker driehoek bedekt. De tijd $t$ wordt gemeten in seconden.
In figuur 3 is de situatie voor een zeker tijdstip $t$ getekend.
Punt $A$ heeft dan de coördinaten $(t, 8)$. Het bedekte gebied is grijs gekleurd.

The distance between A and B at time $t$ is given by $a(t) = \ldots$
Show this.

figuur 3

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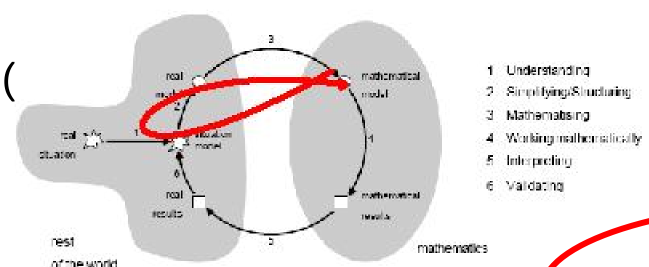



De afstand in cm tussen $A$ en $B$ op tijdstip $t$ noemen we $a(t)$.
Er geldt: $a(t) = \sqrt{128 - 16t + t^2}$ .

3p **6** □ Toon dit aan.

# Are there big discrepancies to actual classroom practice?

- What is *“big”* ?

Not big:

- Assessment is driving
- Many teachers attend “NWD” (a fair on math applications)

Yes, big:

- Teachers’ dependency on textbooks
- Mechanistic modelling (drill of atomistic modelling)
- Math time being cut (time tables)

# What role - if any - has the international modelling community played in the developments in your country, and which role could it play?

- The other way around: the role of Dutch leaders in the international community

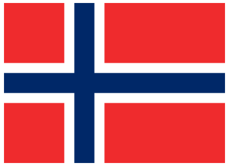
  Internationally frequently cited:
  Hans Freudenthal, Jan de Lange, Marja van den Heuvel-Panhuizen, Koeno Gravemeijer and others

- many design studies

  Adri Dierdorp (2012) - Learning statistical modelling based on authentic practices

- few evidence-based evaluations, few cognitive studies

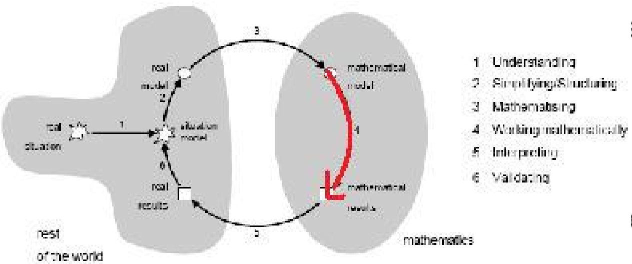
# What is the role of modelling in the curriculum of your country, including assessment?



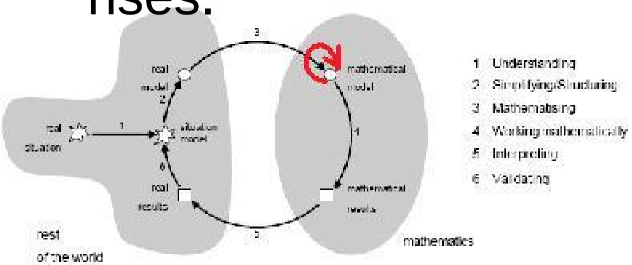
- Virtually inexistent in curriculum documents (terms used: relevance, engaging, attractive, inquiry-based)

- National exams consist of 2 parts
  1st part: without tools, classical “barren” maths
  2nd part: with tools (calc, CAS, off-line computer) more contextualised, never a real problem being solved

When will the saturation be more than 0,8?




Use the derivative to show that the saturation rises when the pressure rises.




## Oppgave 2 (6 poeng)

I lungene blir oksygen bundet til hemoglobin og transportert rundt i kroppen av blodet. Hemoglobinet er mettet når det ikke er i stand til å ta opp mer oksygen.

Den engelske fysiologen A. V. Hill oppdaget i 1910 en sammenheng mellom deltrykket til oksygenet i lungene og metningsgraden $g$ .

A.V. Hill (1886 – 1977)

Nobelprisvinner 1922
(fysiologi eller medisin)

an fant at under visse forhold er

$$g(x) = \frac{x^3}{x^3 + 25000} \quad , \quad x > 0$$

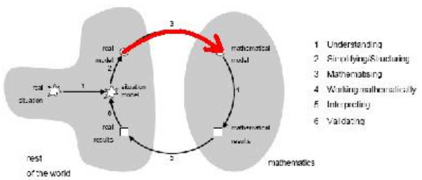
er er deltrykket $x$ målt i mmHg (millimeter kvikksølv).

a) Bruk graftegner til å tegne grafen til $g$.

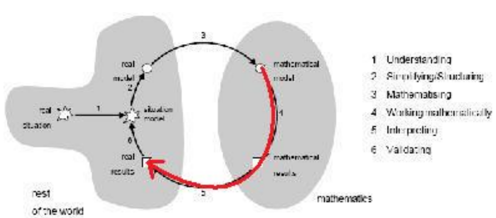
b) Bestem grafisk hva deltrykket $x$ må være for at metningsgraden $g(x)$ skal være større enn 0,8.

c) Bruk derivasjon til å vise at metningsgraden øker dersom deltrykket øker. Forklar.

Use regression to create the models A(x) and B(x) for the relation between windspeed and effect.



For which wind speed have the windmills the same effect?



## Oppgave 4 (8 poeng)

For å få strøm på hytta ønsker Kristian å kjøpe en liten vindmølle. Han vurderer to ulike typer, vindmølle A og vindmølle B. Tabellene nedenfor viser sammenhengen mellom vindhastigheten og den effekten vindmøllene gir.

**Vindmølle A**

| Vindhastighet, $x$ (m/s) | 4,2 | 6,7 | 10,6 | 13,9 |
|---|---|---|---|---|
| Effekt, $A(x)$ (W) | 50 | 100 | 200 | 300 |

**Vindmølle B**

| Vindhastighet, $x$ (m/s) | 5,6 | 7,5 | 10,0 | 11,8 |
|---|---|---|---|---|
| Effekt, $B(x)$ (W) | 50 | 100 | 200 | 300 |

Begge vindmøllene trenger en vindhastighet på minimum 2 m/s for å gå rundt. Dersom vindhastigheten blir større enn 15 m/s, kobler vindmøllene automatisk ut for ikke å bli ødelagt.

Undersøkelser har vist at sammenhengen mellom vindhastigheten og vindmøllenes effekt med god tilnærming kan beskrives ved hjelp av potensfunksjoner.

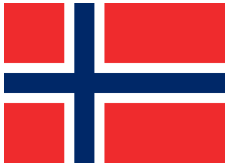
a) Bruk regresjon til å bestemme matematiske modeller $A(x)$ og $B(x)$ som viser sammenhengen mellom vindhastighet og effekt for de to vindmøllene.

b) Tegn grafene til de to modellene innenfor modellenes gyldighetsområde.

c) Ved hvilken vindhastighet gir begge vindmøllene like stor effekt ifølge modellene i oppgave a)?
Hvor mange watt (W) produserer hver av vindmøllene da?

d) Hvor mye større effekt gir vindmølle B sammenliknet med vindmølle A når vindstyrken er 14 m/s? Gi svaret både i watt og i prosent.

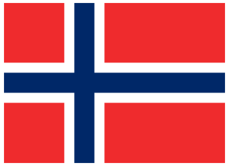
# Are there big discrepancies to actual classroom practice?



- What is *“big”* ?

- Teachers and students prefer to prepare for the 1st part of the exams
- 2nd part of exams is relatively new, teachers not at ease with use of CAS

# What role - if any - has the international modelling community played in the developments in your country, and which role could it play?



- Staff mobility within Nordic countries

  for example
  Morten Blomhøj – Univ of Tromsø
  Mette Andresen – Univ of Bergen
  Mogens Niss – Univ of Agder project

- Seminars with international experts

  2914 Seminar on Vocational Education with G Wake, R Sträßer

  2015 Mathematical Modelling Colloquium with M Blomhøj, A Heck, P Hernandez-Martinez, Th Lingefjärd.

- A need for research

  Suela Kacerja (2012) - ”Real-life contexts in mathematics and students´ interests. An Albanian study”

  Anne Fyhn (2007). Angles as Tool for Grasping Space: Teaching of Angles Based on Students' Experiences with Physical Activities and Body Movement.

| | Contextualised | Not contextualised |
|---|---|---|
| Inquiry-based | Genuine modelling | For example: Euclidean proofs |
| Not Inquiry-based | Inauthentic word problems | For example: algebra drill tasks |

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